Manor Hill First School

Calculation Policy

About our Calculation Policy

This policy has been designed in accordance with the National Curriculum 2014 and helps to develop the three main aims of **Fluency, Reasoning** and **Problem Solving.** It is designed to give pupils a consistent and smooth progression of learning when using the four main operations.

Please note that early learning teaching in number and calculations in Reception follows the EYFS document.

The calculation policy is organised according to age and stage expectations as set out in the National Curriculum 2014. However, it is vital that pupils are taught according to the stage that they are currently working at, moving on when they are secure.

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of the calculation and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority in Maths lessons.

Aims of the written calculation policy

- To support greater consistency in the teaching of written calculations across school.
- To strengthen continuity and progression in children's understanding of the development of written calculations.
- To form a core set of methods which every child will experience and build upon.
- To build on models and images introduced to promote conceptual understanding.
- To provide reference and guidance on the teaching of calculation skills for teaching staff, teaching assistants and parents.

Good Practice in Calculation

- Establish mental methods based on good understanding of place value in numbers and tables facts.
- Show children how to set out written calculations vertically, initially using expanded layouts (starting with adjustments of 'carrying' and introducing this adjustment slowly and systematically.
- Link practical, mental and written methods.
- Make strong links between inverse operations of addition/subtraction and multiplication/division.
- Make sure the children always look out for special cases that can still be done entirely mentally.
- Gradually refine the written record into a more compact standard method.
- Extend to large numbers and decimals.
- Ensure that the understanding of remainders and what to do with them in context, is taught alongside division throughout.
- Once written methods are introduced, keep mental skills sharp by continuing to develop and apply them to appropriate examples. (Encourage children to try mental methods first.)
- Encourage children to identify the best method and make choices.
- Encourage children to use tools to support their learning e.g. Number lines, 100 squares, until they are secure.

Solving a calculation

Read the question

Understand the question

Choose an operation

Solve the question

Answer the question

Check your answer

PRACTICAL → **VISUAL** → **WRITTEN RECORDING**

Year 1 Addition and Subtraction

Statutory requirements

Pupils should be taught to:

- read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs

- represent and use number bonds and related subtraction facts within 20

- add and subtract one-digit and two-digit numbers to 20, including zero

- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$.

Notes and guidance (non-statutory)

Pupils memorise and reason with number bonds to 10 and 20 in several forms (for example, 9 + 7 = 16; 16 – 7 = 9; 7 = 16 – 9). They should realise the effect of adding or subtracting zero. This establishes addition and subtraction as related operations.

Pupils combine and increase numbers, counting forwards and backwards.

They discuss and solve problems in familiar practical contexts, including using quantities. Problems should include the terms: put together, add, altogether, total, take away, distance between, difference between, more than and less than, so that pupils develop the concept of addition and subtraction and are enabled to use these operations flexibly.

Year 1 Add with numbers up to 20

Use numbered number lines to add, by counting on in ones. Encourage children to start with the **larger** number and count on.

$$6 + 3 = 9$$
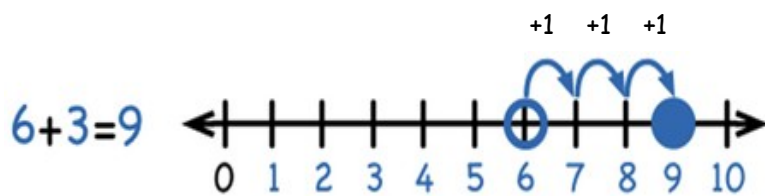
Children should:

- Have access to a wide range of counting equipment, everyday objects, number tracks and number lines, and be shown numbers in different contexts.

- Read and write the addition (+) and equals (=) signs within number sentences.

- Interpret addition number sentences and solve missing box problems, using concrete objects and number line addition to solve them: 8 + 3 = ⬤
  15 + 4 = ⬤   5 + 3 + 1 = ⬤   ⬤ + ⬤ = 6

This builds on from prior learning of adding by combining two sets of objects into one group (5 cubes and 3 cubes) in Early Years.

Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.

8 + 5

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

Key skills for addition at Y1:

- Read and write numbers to 100 in numerals, including 1—20 in words.
- Recall bonds to 10 and 20, and addition facts within 20.
- Count to and across 100.
- Count in multiples of 1 2, 5 and 10.
- Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations.

Year 2 Addition and Subtraction

Statutory requirements

Pupils should be taught to:

- solve problems with addition and subtraction:
    - using concrete objects and pictorial representations, including those involving numbers, quantities and measures
    - applying their increasing knowledge of mental and written methods
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
    - a two-digit number and ones
    - a two-digit number and tens
    - two two-digit numbers
    - adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

Notes and guidance (non-statutory)

Pupils extend their understanding of the language of addition and subtraction to include sum and difference.

Pupils practise addition and subtraction to 20 to become increasingly fluent in deriving facts such as using $3 + 7 = 10$; $10 - 7 = 3$ and $7 = 10 - 3$ to calculate $30 + 70 = 100$; $100 - 70 = 30$ and $70 = 100 - 30$. They check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (for example, $5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5$). This establishes commutativity and associativity of addition.
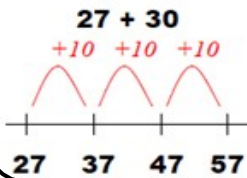
Recording addition and subtraction in columns supports place value and prepares for formal written methods with larger numbers.
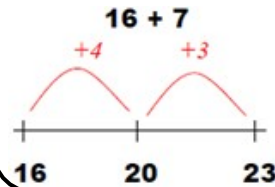
Addition

Year 2 Add with 2-digit numbers
Developing mental fluency with addition and place value involving 2-digit numbers, then establish more formal methods.
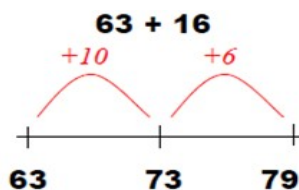
Add 2-digit numbers and tens:

27 + 30
+10 +10 +10

27 37 47 57

Add 2-digit numbers and units:

16 + 7
+4 +3

16   20   23

Use empty number lines, concrete equipment, hundred squares etc. to build confidence and fluency in mental addition skills.

Add pairs of 2-digit numbers, moving to the partitioned column method when secure adding tens and units:

63 + 16
+10   +6

63   73   79

23 + 34:

```
  2 0 + 3
+ 3 0 + 4
  5 0 + 7
      = 5 7
```

STEP 1: Only provide examples that do **NOT** cross the tens boundary until they are secure with the method itself.

STEP 2: Once children can add a multiple of ten to a 2-digit number mentally (e.g. 80+11), they are ready for adding pairs of 2-digit numbers that DO cross the tens boundary (e.g. 58 + 43).

58 + 43:

```
  5 0 + 8
  4 0 + 3
  9 0 + 1 1
        = 1 0 1
```

STEP 3: Children who are confident and accurate with this stage should move onto the partitioned addition methods with 2 and 3-digit numbers (see Y3).

To support understanding, pupils may physically make and carry out the calculation with Dienes Base 10 apparatus or place value counters, then compare their practical version to the written form, to help them to build an understanding of it.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

Key skills for addition at Y2:
- Add a 2-digit number and ones (e.g. 27 + 6).
- Add a 2-digit number and tens (e.g. 23 + 40).
- Add pairs of 2-digit numbers (e.g. 35 + 47).
- Add three single-digit numbers (e.g. 5 + 9 + 7).
- Show that adding can be done in any order (the commutative law).
- Recall bonds to 20 and bonds of tens to 100 (30 + 70 etc.).
- Count in steps of 2, 3 and 5 and count in tens from any number.
- Understand the place value of 2-digit numbers (tens and ones).
- Compare and order numbers to 100 using < > and = signs.
- Read and write numbers to at least 100 in numerals and words.
- Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.

Year 3 Addition and Subtraction

Statutory requirements

Pupils should be taught to:

- add and subtract numbers mentally, including:
    - a three-digit number and ones
    - a three-digit number and tens
    - a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Notes and guidance (non-statutory)

Pupils practise solving varied addition and subtraction questions. For mental calculations with two-digit numbers, the answers could exceed 100.

Pupils use their understanding of place value and partitioning, and practise using columnar addition and subtraction with increasingly large numbers up to three digits to become fluent (see Mathematics Appendix 1).

Addition

Year 3 Add numbers with up to 3-digits

Expand the **partitioned column addition** method to hundreds, tens and units:

> Partition the numbers. Then, using the columns, add the units, tens then hundreds.

In order to carry out this method of addition:

- Children need to recognise the value of the hundreds, tens and units by recording the partitioning.

- Pupils need to be able to add in columns.

Move to the compact **column addition** method, with carrying":

```
  236
+  73
-----
  309
1
```

> Add **units** first.

> "Carry" numbers **underneath** the bottom line.

Children who are very secure and confident with 3-digit partitioned column addition should be moved onto the **compact column addition** method, being introduced to „carrying" for the first time. Compare the partitioned method to the compact column method to develop an understanding of the process and the reduced number of steps involved.

> Remind pupils the actual value is ‚**three tens** add **seven tens**', which equals **ten** tens.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, **hundreds boundary, increase, vertical, carry, expanded, compact**

Key skills for addition at Y3:
- Read and write numbers to 1000 in numerals and words.
- Add 2-digit numbers mentally, including those exceeding 100.
- **Add a three-digit number and ones mentally (175 + 8).**
- **Add a three-digit number and tens mentally (249 + 50).**
- **Add a three-digit number and hundreds mentally (381 + 400).**
- Estimate answers to calculations, using inverse to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- Recognise place value of each digit in 3-digit numbers (hundreds, tens, ones.)
- Continue to practise a wide range of mental addition strategies, ie. number bonds, adding the nearest multiple of 10, 100, 100 and adjusting, using near doubles, partitioning and recombining.

Year 4 Addition and Subtraction

Statutory requirements

Pupils should be taught to:

- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate

- estimate and use inverse operations to check answers to a calculation

- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

1.   Notes and guidance (non-statutory)

Pupils continue to practise both mental methods and columnar addition and subtraction with increasingly large numbers to aid fluency (see Mathematics Appendix 1).

Addition

Year 4 Add numbers with up to 4 digits

Move from partitioned column addition to the compact column method, **adding units first**, and carrying numbers **underneath** the calculation. Also include money and measure contexts.

e.g. 3517 + 396 = 3913

Introduce the **compact column addition** method by asking children to add the two given numbers together using the method that they are familiar with (partitioned column addition—see Y3). Teacher models the compact method with carrying, asking children to discuss similarities and differences and establish how it is carried out.

Add **units** first.

„Carry" numbers **underneath** the bottom line.

Reinforce correct place value by reminding them the actual value is 5 hundreds add 3 hundreds, **not 5 add 3**, for example.

Use and apply this method to money and measurement values.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, „carry", expanded, compact, **thousands, hundreds, digits, inverse**

Key skills for addition at Y4:

- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of each digit in a four-digit number.
- Round any number to the nearest 10, 100 or 1000.
- Estimate and use inverse operations to check answers.
- Solve 2-step problems in context, deciding which operations and methods to use and why.
- Find 1000 more or less than a given number.
- Continue to practise a wide range of mental addition strategies, ie. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining.
- Add numbers with up to 4 digits using the formal written method of column addition.
- Solve 2-step problems in contexts, deciding which operations and methods to use and why.
- Estimate and use inverse operations to check answers to a calculation.

Year 5 Addition and Subtraction

2. Statutory requirements

Pupils should be taught to:

- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)

- add and subtract numbers mentally with increasingly large numbers

- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy

- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

1. Notes and guidance (non-statutory)

Pupils practise using the formal written methods of columnar addition and subtraction with increasingly large numbers to aid fluency (see Mathematics Appendix 1).

They practise mental calculations with increasingly large numbers to aid fluency (for example, 12 462 – 2300 = 10 162).

Addition

Year 5 Add numbers with more than 4 digits

including money, measures and decimals with different numbers of decimal places.

$$
\begin{array}{r}
£23.59 \\
+ £7.55 \\
\hline
£31.14
\end{array}
$$

The decimal point should be aligned in the same way as the other place value columns, and must be in the same column in the answer.

$$
\begin{array}{r}
23,481 \\
+ 1,362 \\
\hline
24,843
\end{array}
$$

Numbers should exceed 4 digits.

Pupils should be able to add more than two values, carefully aligning place value columns.

$$
\begin{array}{r}
19.01 \\
3.65 \\
+ 0.70 \\
\hline
23.36
\end{array}
$$

Say "6 tenths add 7 tenths" to reinforce place value.

Empty decimal places can be filled with zero to show the place value in each column.

Children should:

- Understand the place value of **tenths and hundredths** and use this to align numbers with different numbers of decimal places.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, „carry", expanded, compac**t**, vertical, thousands, hundreds, digits, inverse & **decimal places, decimal point, tenths, hundredths, thousandths**

Key skills for addition at Y5:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies ie. add the nearest multiple of 10, 100, 100 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in contexts, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000.
- Add numbers with more than 4 digits using formal written method of column addition.

Year 6 Addition, Subtraction, Multiplication and Division

Statutory requirements

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

- perform mental calculations, including with mixed operations and large numbers

- identify common factors, common multiples and prime numbers

- use their knowledge of the order of operations to carry out calculations involving the four operations

- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

- solve problems involving addition, subtraction, multiplication and division

- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Notes and guidance (non-statutory)

Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division (see Mathematics Appendix 1).

They undertake mental calculations with increasingly large numbers and more complex calculations.

Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.

Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.

Pupils explore the order of operations using brackets; for example, $2 + 1 \times 3 = 5$ and $(2 + 1) \times 3 = 9$.

Common factors can be related to finding equivalent fractions.

Addition

Year 6 Add several numbers of increasing complexity

$$
\begin{array}{r}
2\,3 . 3\,6\,1 \\
9 . 0\,8 \\
5\,9 . 7\,7 \\
+\quad 1 . 3 \\
\hline
9\,3 . 5\,1\,1 \\
\end{array}
$$

Adding several numbers with different numbers of decimal places (including money and measures):

- Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row.

- Zeros could be added into any empty decimal places, to show there is no value to add.

> Empty decimal places can be filled with zero to show the place value in each column.

$$
\begin{array}{r}
8\,1,0\,5\,9 \\
3,6\,6\,8 \\
1\,5,3\,0\,1 \\
+\ 2\,0,5\,5\,1 \\
\hline
1\,2\,0,5\,7\,9 \\
\end{array}
$$

Adding several numbers with more than 4 digits.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, „carry", expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

Key skills for addition at Y6:
- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.
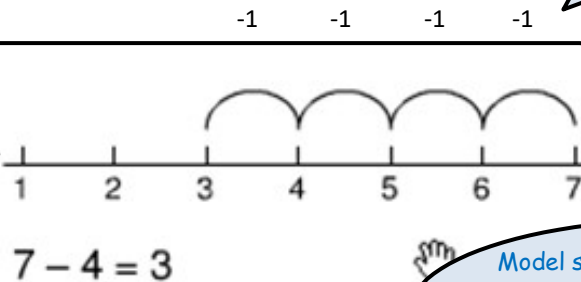
Subtraction

Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes etc. and in familiar contexts, and are introduced to more formal recording using number lines as below:

Read, write and interpret number sentences with **-** and **=** signs.

Subtract by taking away

Count back in ones on a numbered number line to take away, with numbers up to 20:

-1   -1   -1   -1

$7 - 4 = 3$

Model subtraction using hundred squares and numbered number lines/tracks and practically.

Find **the** distance between

This will be introduced practically with the language **'find the distance between'** and **'how many more?'** in a range of familiar contexts.
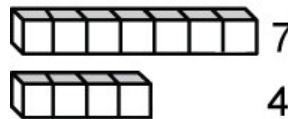
Seven is 3 more than four

7

4

I am 2 years older than my sister"

Mental subtraction

Children should start recalling subtraction facts up to **and within** 10 and 20, and should be able to subtract zero.

Key vocabulary: equal to, take, take away , less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_?

Key skills for subtraction at Y1:
- Given a number, say **one more or one less**.
- Count to and over 100, **forward and back**, from any number.
- Represent and use **subtraction facts to 20 and within 20**.
- Subtract with **one-digit and two-digit** numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects (ie bead string, objects, cubes) and pictures, and missing number problems.
- Read and write numbers from 0 to 20 in numerals and words.

Subtraction

Year 2 Subtract with 2-digit numbers

Subtract on a number line by <u>counting back</u>, aiming to develop mental subtraction skills.
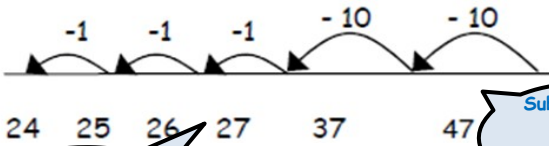
This strategy will be used for:

- **2-digit numbers subtract units** (by taking away / counting back) e.g. 36—7
- **2-digit numbers subtract tens** (by taking away / counting back) e.g. 48—30
- **Subtracting pairs of 2-digit numbers** (see below:)

Use Dienes blocks for subtraction calculations too.

Subtracting pairs of 2-digit numbers on a number line:

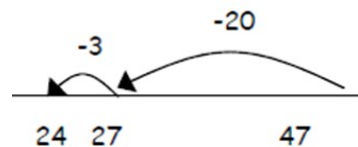**47 - 23 = 24** Partition the second number and subtract it in tens and units, as below:

Move towards more efficient jumps back, as below:

-1 -1 -1 - 10 - 10

24 25 26 27 37 47

Subtract tens first.

Then subtract units.

-3 -20

24 27 47

Combine methods with use of a hundred square to reinforce understanding of number value and order.

Teaching children to **bridge through ten** can help them to become more efficient, for example 42—25:

-3 -2 -20

17 20 22 42
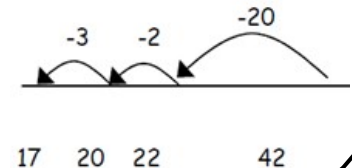
Mental strategy – subtract numbers close together by **counting on**:

Start with the smaller number and count on to the largest.

42 − 38 = 4

+1 +1 +1 +1

37 38 39 40 41 42
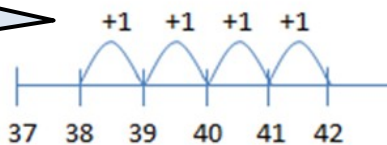
Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to **count on** the difference. They need to be clear about the relationship between addition and subtraction.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units

Key skills for subtraction at Y2:

- Recognise the place value of each digit in a two-digit number.
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two digit number and ones, a two-digit number and tens, and two two-digit numbers.
- Show that subtraction of one number from another cannot be done in any order.
- Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number problems.
- Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods.
- Read and write numbers to at least 100 in numerals and in words.

Year 3 Subtracting with 2 and 3-digit numbers.

Introduce **partitioned column subtraction** method.

STEP 1: introduce this method with examples where **no exchanging** is required.

$$89 - 35 = \underline{54}$$

$$
\begin{array}{r}
80 + 9 \\
- \ 30 + 5 \\
\hline
50 + 4
\end{array}
$$

When learning to „exchange", explore „partitioning in different ways" so that pupils understand that when you exchange, the **VALUE** is the same ie **72** = 70+2 = 60+12 = 50+22 etc. Emphasise that the **value hasn't changed**, we have just partitioned it in a different way.

STEP 2: introduce „exchanging" through practical subtraction. Make the larger number with Base 10, then subtract 47 from it.

72 - 47

$$
\begin{array}{r}
60 \\
7\!\!\!/0 + \overset{1}{2} \\
- \ 40 + \\
7 \\
\hline
20 + 5 \quad = \ \underline{25}
\end{array}
$$

Before subtracting '7' from the 72 blocks, they will need to exchange a row of 10 for ten units. Then subtract 7, and subtract 4 tens.

STEP 3: Once pupils are secure with the understanding of "exchanging", they can use the partitioned column method to subtract any 2 and 3-digit numbers.

$$
\begin{array}{c}
2\ 3\ 8\ -\ 1\ 4\ 6 = 9\ 2 \\[4pt]
\overset{1\ 0\ 0}{2\!\!\!/0\!\!\!/0} + \overset{1}{3}0 + 8 \\
- \quad 1\ 0\ 0 + 4\ 0 + 6 \\
\hline
0 + 9\ 0 + 2
\end{array}
$$

Subtracting money: partition into e.g. £1 + 30p + 8p

Counting on as a mental strategy for subtraction:

Because counting on in tens is the way we use a 100 square.

+10   +3
89   99   102

Continue to reinforce counting **on** as a strategy for **close-together numbers** (e.g. 121—118), and also for numbers that are "nearly" multiples of 10, 100, 1000 or £s, which make it easier to count on (e.g. 102-89, 131—79, or calculating change from £1 etc.).

• Start at the smaller number and count on in **tens** first, then count on in units to find the rest of the difference:
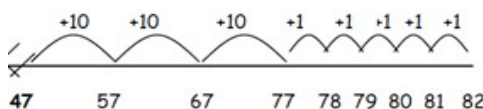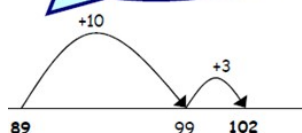
+10 +10 +10   +1 +1 +1 +1 +1
47   57   67   77 78 79 80 81 82

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units **exchange, decrease, hundreds, value, digit**

Key skills for subtraction at Y3:
- Subtract mentally a: **3-digit number and ones, 3-digit number and tens, 3-digit number and hundreds** .
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise the place value of each digit in a 3-digit number .
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10 (see examples above).
- Read and write numbers up to 1000 in numerals and words.
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or 21), and select most appropriate methods to subtract, explaining why.

<u>Year 4</u> Subtract with up to 4-digit numbers

Partitioned column subtraction with exchanging (decomposition):

| 2 | 7 | 5 | 4 | – | 1 | 5 | 6 | 2 | = | 1 | 1 | 9 | 2 |

	2	0	0	0	+	~~7~~ ⁶⁰⁰ 0 0	+	'5	0	+	4
–	1	0	0	0	+	5	0	0	+	6	0	+	2
	1	0	0	0	+	1	0	0	+	9	0	+	2

> As introduced in Y3, but moving towards more complex numbers and values. Use **place value counters** to reinforce exchanging.

> Subtracting money: partition into £1 + 30 + 5 for example.

Compact column subtraction (see video)

```
    6
  2 X '5 4
-  1 5 6 2
   1 1 9 2
```

> To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask pupils to consider how it relates to the method they know, what is similar and what is different, to develop an understanding of it (shown on video).

> Give plenty of opportunities to apply this to money and measures.

> Always encourage children to consider the best method for the numbers involved— mental, counting on, counting back or written method.

Mental strategies

A variety of mental strategies must be taught and practised, including counting on to find the difference where numbers are closer together, or where it is easier to count on.

<u>Key vocabulary</u>: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is _? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse

Key skills for subtraction at Y4:

- Subtract by counting on or back where numbers are close together or they are near to multiples of 10, 100 etc.
- Children select the most appropriate and efficient methods for given subtraction calculations.
- Estimate and use inverse operations to check answers.
- Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why.
- Solve simple measure and money problems involving fractions and decimals to two decimal places.
- Find 1000 more or less than a given number.
- Count backwards through zero, including negative numbers.
- Recognise place value of each digit in a 4-digit number Round any number to the nearest 10, 100 or 1000
- Solve number and practical problems that involve the above, with increasingly large positive numbers.

Year 5 Subtract with at least 4-digit numbers

including money, measures, decimals.

Compact column subtraction

(with "exchanging").

> Children who are still not secure with number facts and place value will need to remain on the partitioned column method until ready for the compact method.

Subtracting with larger integers.

Subtract with decimal values, including mixtures if integers and decimals, aligning the decimal point.

> Add a "zero" in any empty decimal places to aid understanding of what to subtract in that column.

> Create lots of opportunities for subtracting and finding differences with money and measures.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, exchange, decrease, hundreds, value, digit, inverse, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units, **tenths, hundredths, decimal point, decimal**

Key skills for subtraction at Y5:

- Subtract numbers mentally with increasingly large numbers .
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy .
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Count forwards or backwards in steps of powers of 10 for any given number up to 1 million.
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0.
- Round any number up to 1 million to the nearest 10, 100, 1000, 10 000 and 100 000.

Year 6 Subtracting with increasingly large and more complex numbers and decimal values.

```
  ° ₁ ¹ ⁹
  X  8  10,  6  9  9
-     8  9,  9  4  9
  ─────────────────
     6  0,  7  5  0
```

Using the compact column method to subtract more complex integers.

```
  ° ₁ ⁵   ³   ¹
  X  10  5 ·  4  1  9  kg
-     3  6 ·  0  8  0  kg
  ──────────────────────
     6  9 ·  3  3  9  kg
```

Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.

Empty decimal places can be filled with **zero** to show the place value in each column.

Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting **the most appropriate method** to work out subtraction problems.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back , how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal

Key skills for subtraction at Y6:

- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy.
- Use negative numbers in context, and calculate intervals across zero.
- Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate.

Year 1 Multiplication and Division

Statutory requirements

Pupils should be taught to:

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Notes and guidance (non-statutory)

Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of objects, numbers and quantities.

They make connections between arrays, number patterns, and counting in twos, fives and tens.
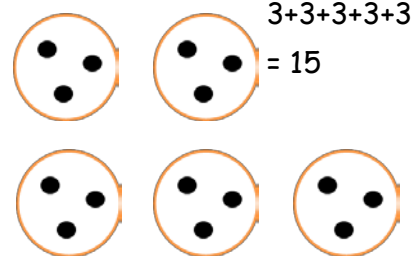
Multiplication

How many legs will 3 teddies have?

2 + 2 + 2 = 6

There are 3 sweets in one bag. How many sweets are in 5 bags altogether?

3+3+3+3+3
= 15

- Give children experience of counting equal groups of objects in 2s, 5s and 10s.

- Present practical problem solving activities involving counting equal sets or groups, as above.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count

Key skills for multiplication at Y1:

- Count in multiples of 2, 5 and 10.
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Make connections between arrays, number patterns, and counting in twos, fives and tens. Begin to understand doubling using concrete objects and pictorial representations.

Year 2 Multiplication and Division

Statutory requirements

Pupils should be taught to:

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers

- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs

- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot

- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Notes and guidance (non-statutory)

Pupils use a variety of language to describe multiplication and division.

Pupils are introduced to the multiplication tables. They practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other. They connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. They begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations.

Pupils work with a range of materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition. They begin to relate these to fractions and measures (for example, 40 ÷ 2 = 20, 20 is a half of 40). They use commutativity and inverse relations to develop multiplicative reasoning (for example, 4 × 5 = 20 and 20 ÷ 5 = 4).

Multiplication

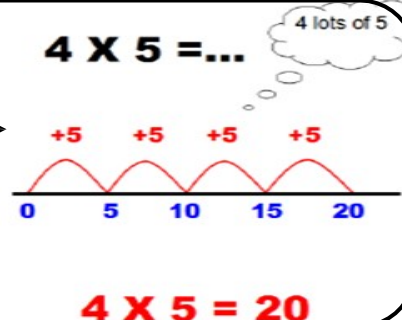## Year 2 Multiply using arrays and repeated addition

(using at least 2s, 5s and 10s)

$4 \times 5 = ...$

4 lots of 5

Use repeated addition on a number line:

- Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and = signs.

+5 +5 +5 +5

0 5 10 15 20

$4 \times 5 = 20$

Use arrays:

$5 \times 3 = 15$

$3 \times 5 = 15$

$5 \times 3 = 3 + 3 + 3 + 3 + 3 = \underline{15}$

$3 \times 5 = 5 + 5 + 5 = \underline{15}$

Use arrays to help teach children to understand the commutative law of multiplication, and give examples such as $3 \times \underline{\ \ } = 6$.

$5 \times 3 = 5 + 5 + 5$

Use practical apparatus:

5   5   5
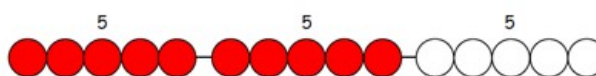
Use mental recall:

- Children should begin to **recall multiplication facts for 2, 5 and 10** times tables through practise in counting and understanding of the operation.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times...

Key skills for multiplication at Y2:
- Count in steps of 2, 3 and 5 from zero, and in 10s from any number.
- Recall and use multiplication facts from the **2, 5 and 10** multiplication tables, including recognising odds and evens.
- Write and calculate number statements **using the x and = signs**.
- Show that multiplication can be done in any order (commutative).
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods, and multiplication facts.
- Pupils use a variety of language to discuss and describe multiplication.

Year 3 Multiplication and Division

Statutory requirements

Pupils should be taught to:

- recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables

- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Notes and guidance (non-statutory)

Pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables.

Pupils develop efficient mental methods, for example, using commutativity and associativity (for example, $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and multiplication and division facts (for example, using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts (for example, $30 \times 2 = 60$, $60 \div 3 = 20$ and $20 = 60 \div 3$).

Pupils develop reliable written methods for multiplication and division, starting with calculations of two-digit numbers by one-digit numbers and progressing to the formal written methods of short multiplication and division.

Pupils solve simple problems in contexts, deciding which of the four operations to use and why. These include measuring and scaling contexts, (for example, four times as high, eight times as long etc.) and correspondence problems in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

Multiplication

Introduce the grid method for multiplying 2-digit by single-digits:

Eg.   23 x 8 = 184

X	20	3
8	160	24

160 + 24 = 184

Link the layout of the grid to an array initially:
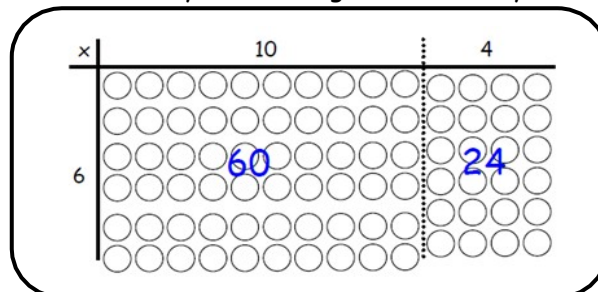
Introduce the grid method with children physically making an array to represent the calculation (e.g. make 8 lots of 23 with 10s and 1s place value counters), then translate this to grid method format.

To do this, children must be able to:

- Partition numbers into tens and units.
- Multiply multiples of ten by a single digit (e.g. 20 x 4) using their knowledge of multiplication facts and place value.
- Recall and work out multiplication facts in the **2, 3, 4, 5, 8 and 10** times tables.
- Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays:

9 x 4 = 36
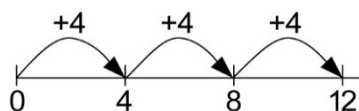
Key skills for multiplication:

- Recall and use multiplication facts for the **2, 3, 4, 5, 8 and 10** multiplication tables, and multiply multiples of 10.
- Write and calculate number statements using the multiplication tables they know, including **2-digit x single-digit**, drawing upon mental methods, and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.
- Develop mental strategies using commutativity (e.g. 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240).
- Solve simple problems in contexts, deciding which operations and methods to use.
- Develop efficient mental methods to solve a range of problems e.g using commutativity (**4 × 12 × 5** = 4 × 5 × 12 = **20 × 12** = 240) and for missing number problems   x 5 = 20,   3 x   = 18,   x   = 32

Year 4 Multiplication and Division

Statutory requirements

Pupils should be taught to:

- recall multiplication and division facts for multiplication tables up to 12 × 12

- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers

- recognise and use factor pairs and commutativity in mental calculations

- multiply two-digit and three-digit numbers by a one-digit number using formal written layout

- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Notes and guidance (non-statutory)

Pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency.

Pupils practise mental methods and extend this to three-digit numbers to derive facts, (for example $600 \div 3 = 200$ can be derived from $2 \times 3 = 6$).

Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers (see Mathematics Appendix 1).

Pupils write statements about the equality of expressions (for example, use the distributive law $39 \times 7 = 30 \times 7 + 9 \times 7$ and associative law $(2 \times 3) \times 4 = 2 \times (3 \times 4)$). They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, $2 \times 6 \times 5 = 10 \times 6 = 60$.

Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as the numbers of choices of a meal on a menu, or three cakes shared equally between 10 children.

Multiplication

Year 4 Multiply 2 and 3-digits by a single digit, using all multiplication tables up to 12 x 12

Developing the grid method:

Eg. 136 x 5 = 680

X	100	30	6
5	500	150	30

```
  500
  150
+  30
  680
```

Encourage column addition to add accurately.

Move onto short multiplication (see Y5) if and when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit this way, and are already confident in "carrying" for written addition.

Children should be able to:

- **Approximate before they calculate**, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer. e.g:

  –346 x 9 is approximately 350 x 10 = **3500**
  Record an approximation to check the final answer against.

- Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge.

- Recall all times tables **up to 12 x 12.**

Key vocabulary: ~~groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated~~ addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times as big as, once, twice, three times... partition, grid method, total, multiple, product, sets of, **inverse**

Key skills for multiplication at Y4:
- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for **all multiplication tables up to 12 x 12.**
- Recognise place value of digits in up to 4-digit numbers.
- Use place value, known facts and derived facts to multiply mentally (e.g. multiply by 1, 10, 100, by 0) or to multiply 3 numbers.
- Use commutativity and other strategies mentally 3 x 6 = 6 x 3 , 2 x 6 x 5 = 10 x 6 , 39x7 = 30 x 7 + 9 x 7.
- Solve problems with increasingly complex multiplication in a range of contexts.
- Count in multiples of 6, 7, 9, 25 and 1000.
- Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones).

Year 5 Multiplication and Division

Statutory requirements

Pupils should be taught to:

- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers

- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers

- establish whether a number up to 100 is prime and recall prime numbers up to 19

- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers

- multiply and divide numbers mentally drawing upon known facts

- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000

- recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})

- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes

- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign

- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Notes and guidance (non-statutory)

Pupils practise and extend their use of the formal written methods of short multiplication and short division (see Mathematics Appendix 1). They apply all the multiplication tables and related division facts frequently, commit them to memory and use them confidently to make larger calculations.

They use and understand the terms factor, multiple and prime, square and cube numbers.

Pupils interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (for example, $98 \div 4 = \frac{98}{4} = 24 \text{ r } 2 = 24\frac{1}{2} = 24.5 \approx 25$).

Pupils use multiplication and division as inverses to support the introduction of ratio in year 6, for example, by multiplying and dividing by powers of 10 in scale drawings or by multiplying and dividing by powers of a 1000 in converting between units such as kilometres and metres.

Distributivity can be expressed as $a(b + c) = ab + ac$.

They understand the terms factor, multiple and prime, square and cube numbers and use them to construct equivalence statements (for example, 4 x 35 = 2 x 2 x 35; 3 x 270 = 3 x 3 x 9 x 10 = 9^2 x 10).

Pupils use and explain the equals sign to indicate equivalence, including in missing number problems (for example, 13 + 24 = 12 + 25; 33 = 5 x ☐).

Multiplication

Year 5 Multiply up to 4-digits by 1 or 2 digits.

Introducing column multiplication

- Introduce by comparing a grid method calculation to a short multiplication method, to see how the steps are related, but notice how there are less steps involved in the column method.
- Children need to be taught to approximate first, e.g. for **72 x 38, they will use rounding: 72 x 38** is approximately 70 x 40 = **2800**, and use the approximation to check the reasonableness of their answer against.

Short multiplication for multiplying by a single digit

x	300	20	7
4	1200	80	28

$$\begin{array}{r} 327 \\ \times \quad 4 \\ \hline 1308 \\ {\scriptstyle 1 \quad 2} \end{array}$$

Pupils could be asked to work out a given calculation using the grid, and then compare it to „your" column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.

Introduce long multiplication for multiplying by 2 digits

	10	8
10	100	80
3	30	24

$$\begin{array}{r} 18 \\ \times \quad 13 \\ \hline 54 \\ 180 \\ \hline 234 \end{array}$$

18 x 3 on the 1st row

(8 x 3 = 24, carrying the 2 for twenty, then „1" x 3).

18 x 10 on the 2nd row. Put a zero in units first, then say 8 x 1, and 1 x 1.

The grid can be used to introduce long multiplication as the relationship can be seen between the answers in each row.

Moving towards more complex numbers:

$$\begin{array}{r} 1234 \\ \times \quad 16 \\ \hline 7404 \\ 12340 \\ \hline 19,744 \end{array}$$

$$\begin{array}{r} 3652 \\ \times \quad 8 \\ \hline 29,216 \\ {\scriptstyle 5 \; 4 \; 1} \end{array}$$

Key vocabulary groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times..., partition, grid method, total, multiple, product, inverse, **square, factor, integer, decimal, short/long multiplication, carry**

Key skills for multiplication at Y5:
- Identify multiples and factors, using knowledge of **multiplication tables to 12x12.**
- Solve problems where larger numbers are decomposed into their factors.
- Multiply and divide integers and decimals by 10, 100 and 1000.
- Recognise and use square and cube numbers and their notation.
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.

Statutory requirements

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication

- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context

- perform mental calculations, including with mixed operations and large numbers

- identify common factors, common multiples and prime numbers

- use their knowledge of the order of operations to carry out calculations involving the four operations

- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

- solve problems involving addition, subtraction, multiplication and division

- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Notes and guidance (non-statutory)

Pupils practise addition, subtraction, multiplication and division for larger numbers, using the formal written methods of columnar addition and subtraction, short and long multiplication, and short and long division (see Mathematics Appendix 1).

They undertake mental calculations with increasingly large numbers and more complex calculations.

Pupils continue to use all the multiplication tables to calculate mathematical statements in order to maintain their fluency.

Pupils round answers to a specified degree of accuracy, for example, to the nearest 10, 20, 50 etc., but not to a specified number of significant figures.

Pupils explore the order of operations using brackets; for example, $2 + 1 \times 3 = 5$ and $(2 + 1) \times 3 = 9$.

Common factors can be related to finding equivalent fractions.

Multiplication

Year 6 Short and long multiplication as in Y5, and multiply decimals with up to 2d.p by a single digit.

$$
\begin{array}{ccccc}
 & 3 & . & 1 & 9 \\
\times & & & & 8 \\
\hline
2\,5 & . & 5 & 2 \\
\end{array}
$$

Line up the decimal points in the question and the answer.

Remind children that the single digit belongs in the units column.

This works well for multiplying money (£.p) and other measures.

Children will be able to:

- Use rounding and place value to make approximations before calculating and use these to check answers against.

- Use **short multiplication** (see Y5) to multiply numbers with **more than 4-digits by a single digit**; to multiply money and measures, and to **multiply decimals with up to 2d.p. by a single digit.**

- Use **long multiplication** (see Y5) to multiply numbers with **at least 4 digits by a 2-digit number.**

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times, partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short / long multiplication, carry, **tenths, hundredths, decimal**

Key skills for multiplication at Y6:

- Recall multiplication facts for all times tables up to **12 x 12 (as Y4 and Y5).**
- Multiply multi-digit numbers, up to 4-digit x 2-digit using long multiplication.
- Perform mental calculations with mixed operations and large numbers.
- Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.
- Estimate answers using round and approximation and determine levels of accuracy.
- Round any integer to a required degree of accuracy.

Division

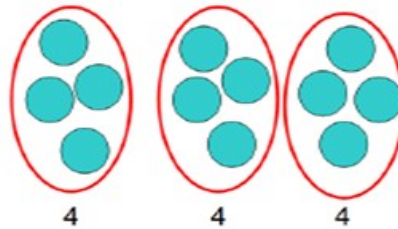## Year 1   Group and share small quantities

Using objects, diagrams and pictorial representations to solve problems involving **both** grouping **and** sharing.

How many groups of 4 can be made with 12 stars? = **3**

Grouping:

Sharing:

4   4   4

12 shared between 3 is 4

Example division problem in a familiar context:

There are 6 pupils on this table and there are 18 pieces of fruit to share between us. If we share them equally, how many will we each get?

Can they work it out and give a division statement… ?

"18 shared between 6 people gives you 3 each."

Pupils should :

- Use lots of practical apparatus, arrays and picture representations

- Be taught to understand the difference between „grouping" objects (How many groups of 2 can you make?) and „sharing" (Share these sweets between 2 people)

- Be able to count in multiples of 2s, 5s and 10s.

- Find **half** of a group of objects by sharing into 2 equal groups.

Key Vocabulary: share, share equally, one each, two each…, group, groups of, lots of, array

**Key number skills needed for division at Y1:**

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher.
- Through grouping and sharing small quantities, pupils begin to understand, division, and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in twos, fives and tens.

Division

Year 2 Group <u>and</u> share, using the ÷ and = sign

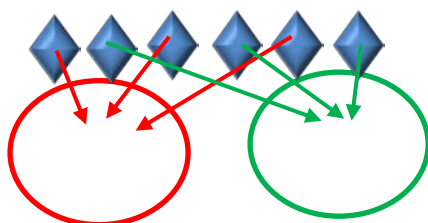Use objects, arrays, diagrams and pictorial representations, and grouping on a number line.

Arrays:
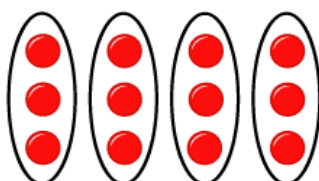
$12 ÷ 3 = 4$

This represents **12 ÷ 3, posed as** how many groups of 3 are in 12?

Pupils should also show that the same array can represent **12 ÷ 4 = 3** if grouped horizontally.

Know and understand sharing <u>and</u> grouping: $6 ÷ 2 = 3$

Sharing
$6 ÷ 2$

Grouping
$6 ÷ 2$

Children should be taught to recognise whether problems require sharing or grouping.

Grouping using a number line:

Group from zero in equal jumps of the divisor to find out "how many groups of _ in _ ?". Pupils could add using a bead string or practical apparatus to work out problems like: A CD costs £3. How many CDs can I buy with £12? **This is an important method to develop understanding of division as grouping.**

+3   +3   +3   +3

1 2 3 4 5 6 7 8 9 10 11 12
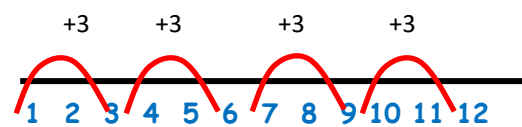
$12 ÷ 3 = 4$

Pose **12 ÷ 3** as „**How many groups of 3 are in 12?**"

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of,
array, divide, divided by, divided into, division, grouping, number line, left, left over

**Key number skills needed for division at Y2:**
- Count in steps of 2, 3, and 5 from 0.
- Recall and use multiplication and division facts for the **2, 5 and 10** multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the x, ÷ and = signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Division

Year 3 Divide 2-digit numbers by a single digit (where there is no remainder in the final answer)

Grouping on a number line:

4 r 1

+3   +3   +3   +3   r1

1 2 3 4 5 6 7 8 9 10 11 12 13
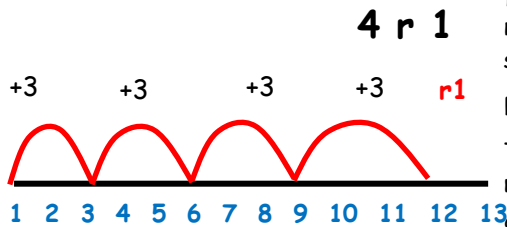
STEP 1: Children continue to work out unknown division facts by grouping on a number line from zero. They are also now taught the concept of remainders, as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s, ready for carrying remainders across within the short division method.

Short division: Limit numbers to NO remainders in the answer OR carried (each digit must be a multiple of the divisor).

3 2
3) 9 6

STEP 2: Once children are secure with division as grouping and demonstrate this using number lines, arrays etc., **short division** for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all. Start by introducing the layout of short division by comparing it to an array.
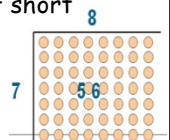
Remind children of correct place value, that 96 is equal to 90 and 6, but in short division, pose:

- How many 3's in 9? = 3, and record it above the **9 tens**.
- How many 3's in 6? = 2, and record it above the **6 units**.

8
7 5 6

Short division: Limit numbers to NO remainders in the final answer, but with remainders occurring within the Calculation.

1 8
4) 7 ³2

STEP 3: Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. 96÷4), and be taught to carry the remainder onto the next digit. **If needed, children should use the number line to work out individual division facts that occur which they are not yet able to recall mentally.**

> *Step 3 Only taught when pupils can calculate remainders.*

Real life contexts need to be used routinely to help pupils gain a full understanding, and the ability to recognise the place of division and how to apply it to problems.

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, **inverse, short division, carry, remainder, multiple**

Key number skills needed for division at Y3:

- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling, connect the 2, 4 and 8s).
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and division.
- Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (30 × 2 = 60, so 60 ÷ 3 = 20 and 20 = 60 ÷ 3).
- Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.

Division
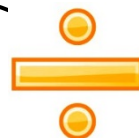
Year 4 Divide up to 3-digit numbers by a single digit
(without remainders initially)

Continue to develop short division:

Short division should only be taught once children have secured the skill of calculating „remainders".

$$4 \overline{)7\,^3 2} = 18$$

STEP 1: Pupils must be secure with the process of short division for dividing 2-digit numbers by a single digit (**those that do not result in a final remainder**—see steps in Y3), but must understand how to calculate remainders, using this to carry remainders within the calculation process.

$$4 \overline{)8\,7\,^3 2} = 218$$

STEP 2: Pupils move onto dividing numbers with up to **3-digits** by a single digit, however problems and calculations provided should **not result in a final answer with remainder** at this stage. Children who exceed this expectation may progress to Y5 level.

Real life contexts need to be used routinely to help pupils gain a full understanding, and the ability to

recognise the place of division and how to apply it to problems.

$$5 \overline{)1\,^1 8\,^3 5} = 037$$

When the answer for the **first column** is zero (1 ÷ 5, as in example), children could initially write a zero above to acknowledge its place, and must always „carry" the number (1) over to the next digit as a remainder.

Include money and measure contexts when confident.

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, „carry",
remainder, multiple, **divisible by, factor**

Key number skills needed for division at Y4:
- **Recall multiplication and division facts for all numbers up to 12 x 12.**
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number.
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example 200 × 3 = 600 so 600 ÷ 3 = 200.
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Division
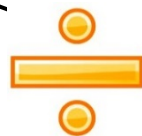
<u>**Year 5**</u> **Divide up to 4 digits by a single digit,** including those with remainders.

Short division, including remainder answers:

$$\begin{array}{r} 0\ 6\ 6\ 3\ r\ 5 \\ 8)\overline{5\,^53\,^50\,^29} \end{array}$$

Short division with remainders: Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where **pupils consider the meaning of the remainder and <u>how</u> to express it,** ie. as a fraction, a decimal, or as a rounded number or value , depending upon the context of the problem.

The answer to 5309 ÷ 8 could be expressed as 663 and five eighths, 663 r 5, as a decimal, or rounded as appropriate to the problem involved.

See Y6 for how to continue the short division to give a decimal answer for children who are confident.

Include money and measure contexts.

If children are confident and accurate:

• Introduce **long division** for pupils who are ready to divide any number by a 2-digit number (e.g. 2678 ÷ 19). This is a Year 6 expectation.

Key Vocabulary: share, share equally, one each, two each…, group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, carry, remainder, multiple, divisible by, factor, inverse, **quotient, prime number, prime factors, composite number (non-prime)**

Key number skills needed for division at Y5:
• Recall multiplication and division facts for all numbers up to 12 x 12 (as in Y4).
• Multiply and divide numbers mentally, drawing upon known facts.
• Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.
• Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
• Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
• Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
• Work out whether a number up to 100 is prime, and recall prime numbers to 19.
• Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.
• Use multiplication and division as inverses.
• Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. 98 ÷ 4 = 24 r 2 = $24\frac{1}{2}$ = 24.5 ≈ 25).
• Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple rates.

Year 6 Divide at least 4 digits by both single-digit and 2-digit numbers (including decimal numbers and quantities)

Division

Short division, for dividing by a single digit: e.g. 6497 ÷ 8

```
   0 8 1 2 · 1 2 5
8)6 4 9⁶7 ·¹0²0⁴0
```

Short division with remainders: Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.

Calculating a decimal remainder: In this example, rather than expressing the remainder as **r 1**, a decimal point is added after the units because there is still a remainder, and the one remainder is carried onto zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.

Introduce long division by chunking for dividing by 2 digits.

```
        27
36 )  972
   -  720      20x
      252
   -  252       7x
        0
              ↓
Answer :      27
```

Find out how many 36s are in 972 by subtracting "chunks" of 36 until zero. Teach pupils to write a '**useful list**' first at the side that will help them decide what chunks to use, e.g.:

Useful' list:
1x = 36
10x = 360
100x = 3600

Introduce the method in a simple way by limiting the choice of chunks e.g Can we use 10 lots? Can use 100 lots? As children become confident with the process, encourage more efficient chunks to get to the answer more quickly (e.g. 20x, 5x), and expand on their "useful" lists.

> Must be aligned in place value for subtracting

> Where remainders occur, pupils should express them as fractions, decimals or use rounding, depending upon the problem.

432 ÷ 15 becomes

```
        2 8 · 8
1 5 ) 4 3 2 · 0
      3 0  ↓
      1 3 2
      1 2 0  ↓
        1 2 0
        1 2 0
            0
```

Answer: 28·8

This is the standard written long division method.
Step 1: 4 ÷ 15
Step 2: 43 ÷ 15 = 2 r 13
Step 3: write 2 on the answer box
Step 4: write 2 x 15 (30) below 43
Step 5: do 43 – 30 = 13
Step 6: write the answer below
Step 7: bring down the next digit (2)
Step 8: 132 ÷ 15 = 8
Step 9: write the 8 on the answer box
Step 10: write 8 x 15 (120) below 132
Step 11: do 132 -120 = 12
Step 12: write . 0 on the end of 432
Step 13: bring down the 0
Step 14: 120 ÷ 15 = 8
Step 15: write . 8 on the answer line
Step 16: write 8 x 15 (120) below 120
Step 17: do 120 – 120 = 0

Key Vocabulary: As previously, & common factor

Key number skills needed for division at Y6:

- Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.